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TITLES AND ABSTRACTS

Macroscopically minimal hypersurfaces

Hannah Alpert

A decades-old application of the second variation formula proves that if the scalar curvature of a closed 3–manifold is bounded below by that of the product of the hyperbolic plane with the line, then every 2–sided stable minimal surface has area at least that of the hyperbolic surface of the same genus. We can prove a coarser analogue of this statement, taking the appropriate notions of macroscopic scalar curvature and macroscopic minimizing hypersurface from Guth's 2010 proof of the systolic inequality for the n–dimensional torus. The appropriate analogue of hyperbolic area in this setting turns out to be the Gromov simplicial norm.

Joint work with Kei Funano.

On groups of isometries preserving multiple horospheres

Grigori Avramidi

Suppose that a discrete group G acts on a Hadamard manifold X by covering space transformations. Let $Fix^0(G)$ be the set of points at infinity whose horospheres are preserved by G. I will discuss the topology of $Fix^0(G)$ and the relation between the (homological) dimension of G and the dimension of $Fix^0(G)$.

Joint work with Tam Nguyen Phan.

Macroscopic scalar curvature and smoothing techniques

Florent Balacheff

In a recent paper L. Guth proved the following theorem : if an hyperbolic manifold endowed with an auxiliary Riemannian metric has sufficiently small volume in comparison to the hyperbolic one, then we can find in its universal cover a ball of radius 1 whose volume is bigger than a hyperbolic ball of radius 1. This theorem is strongly related to a conjecture by R. Schoen, and is conjectured to hold for any radius. In this talk, I will present new results around this topic obtained in collaboration with S. Karam by using a technique due to Gromov and which deserves to be better known : the smoothing method.

Date: April 5, 2019.

Homological norms on nonpositively curved manifolds

Chris Connell

The Gromov-Thurston norm on the singular homology of a closed manifold provides a topological notion of «volume» for a homology class. On the other hand, every such homology class has a dual cohomology class that can be represented by a unique harmonic differential form (with respect to a Riemannian metric) representing that class via the de Rham isomorphism. Forms come equipped with a natural L^2 norm, and the harmonic norm is the L^2 norm of this harmonic form. In joint work with Shi Wang, we relate the Gromov-Thurston norm on homology to the harmonic norm on cohomology with upper and lower bounds that depend (necessarily) on the volume and injectivity radius for nonpositively curved manifolds. This extends work of Brock and Dunfield as well as work of Bergeron, Sengun and Venkatesh. We also will discuss some consequences of this relationship between the norms.

Infinite Volume Rigidity and Bounded Cohomology

James Farre

The bounded cohomology of a group encodes a wealth of geometric and algebraic data. We will define bounded cohomology of groups and construct explicit examples in dimension three; they come from computing the volumes of locally geodesic tetrahedra in hyperbolic manifolds. We use the classification theorems for finitely generated Kleinian groups to prove an infinite volume rigidity result. In particular, we will explain how the volume class of an infinite volume hyperbolic 3-manifold is a complete invariant of its bi-Lipschitz type when restricted to manifolds without parabolic cusps.

Integral Approximation of Simplicial Volume

Daniel Fauser

Stable integral simplicial volume is the stabilization of integral simplicial volume under finite coverings. An oriented closed connected manifold is said to satisfy integral approximation if the stable integral simplicial volume equals the ordinary simplicial volume. The question whether all aspherical manifolds with residually finite fundamental group and vanishing simplicial volume satisfy integral approximation is open. A positive answer would imply a positive answer to the question of Gromov whether all L^2-Betti numbers of an aspherical manifold with vanishing simplicial volume are zero in the case of manifolds with residually finite fundamental group.

We present a short argument for a positive answer to these questions if one assumes that the stable integral simplicial volume was functorial on aspherical manifolds using URC-manifolds by Gaifullin. We also shortly sketch how to prove integral approximation of aspherical smooth manifolds with residually finite fundamental group that admit a non-trivial smooth S^1-action.

Multicomplexes, diffusion of chains and simplicial volume of open manifolds

Roberto Frigerio

In his pioneering paper *Volume and bounded cohomology*, Mikhail Gromov initiated and developed the study of the simplicial volume of closed and open manifolds by making use of his theory of multicomplexes. In the subsequent years, bounded cohomology has been extensively exploited to obtain vanishing results and exact computations for the simplicial volume of closed manifolds. Unfortunately, in the case of open manifolds the duality between ℓ^1-homology and bounded cohomology turns out to be less effective, and one needs to resort to the analysis of concrete cycles. In this talk I will describe how this strategy can be pursued by suitably developing Gromov's theory of multicomplexes.

Joint work with Marco Moraschini.

Stability in bounded cohomology of symplectic lattices

Tobias Hartnick

Bounded cohomology of discrete groups is a powerful tool in geometry and rigidity theory and closely related to simplicial volume. Computing bounded cohomology of groups seems to be a very hard problem, in particular there is still not a single group whose bounded cohomology ring is non-trivial and known. A more tractable problem is to study stable bounded cohomology along families of groups. By a theorem of Monod, bounded cohomology stabilizes along a family of lattices in Lie groups of increasing rank if and only if continuous bounded cohomology of the envelopes stabilizes, and in this case the stable bounded cohomology rings are isomorphic. Moreover, there are precise conjectures about the supposed values that these stable bounded cohomology rings take.

In this talk I will firstly explain the conjectural statements about stable bounded cohomology of lattices and survey what is known towards these conjectures. I will then explain a general method that allows one to obtain stability results for continuous bounded cohomology of Lie groups and explain how it can be applied to the case of linear and symplectic groups. The former is a result of Monod, whereas the latter is a recent joint work with Carlos de la Cruz Mengual. A particular focus will be on explaining the new difficulties which arise in the symplectic case, and which are absent in the case of linear groups.

A vanishing theorem for the bounded cohomology of a foliation with amenable fundamental groupoid

Alessandra Iozzi

We illustrate the proof of a vanishing theorem for the tangential de Rham cohomology of a compact foliated space with amenable fundamental groupoid, by using the existence of bounded primitive of closed bounded differential forms in degree above the rank (for an appropriate notion). In the case of foliated bundles we give a proof of a related theorem asserting the vanishing of the tangential singular cohomology by methods of homological algebra.

Steenrod problem and the domination relation

Jean-François Lafont

I will explain how to combine some classical topology (Thom's work on the Steenrod problem) with some more modern topology (Gromov's simplicial volume) to show that every map between certain manifolds must have degree zero. This is joint work with Christoforos Neofytidis.

Some topological isoperimetric problems

Fedor Manin

Define the complexity of a smooth manifold M to be the least volume of a Riemannian metric on M with certain bounds on the geometry. (I will explain why this is the right notion.) In the 1990's, Gromov asked: what is the relationship between the complexity of a nullcobordant manifold and the complexity of its simplest filling? In recent work with Chambers, Dotterrer, and Weinberger, we have shown an almost-linear bound; in the case of 3–manifolds, this improves upon a result of Costantino and D. Thurston.

This can be thought of as a kind of isoperimetric question for manifolds. I will explain how it relates to notions of isoperimetry in other domains.

Gromov's theory of multicomplexes with applications to closed manifolds

Marco Moraschini

In his pioneering paper *Volume and bounded cohomology* Gromov introduced both the notion of bounded cohomology of topological spaces and simplicial volume of manifolds. His original approach is based on the theory of multicomplexes. Multicomplexes are a simplicial structure which lie in the middle between simplicial complexes and simplicial sets. The aim of this talk is to introduce this «new» simplicial structure and explain why this is the correct approach to the study of bounded cohomology and simplicial volume. More precisely, we will discuss a complete proof of the well-known Gromov's Mapping Theorem via multicomplexes. Beyond bounded cohomology, we will also provide some applications to the simplicial volume of closed manifolds, e.g. Gromov's Vanishing Theorem.

Besides the applications to closed manifolds, the theory of multicomplexes provides useful information on the simplicial volume of open manifolds. These applications will be discussed in the talk of Roberto Frigerio.

Joint work with Roberto Frigerio.

Functorial invariants for circle bundles and for fibrations over the circle

Christoforos Neofytidis

A non-negative numerical quantity I of a closed oriented manifold M is said to be functorial if whenever there is a map $f\colon M \to N$ then $I(M) \geq |\deg(f)|I(N)$. A prominent example is given by Gromov's simplicial volume. We will discuss vanishing and non-vanishing results of functorial numerical invariants for two classes of manifolds: In one direction, we show that there are non-vanishing functorial numerical invariants on each non-trivial circle bundle over an aspherical manifold with hyperbolic fundamental group, generalising thus in all dimensions a result of Brooks and Goldmann for circle bundles over hyperbolic surfaces. In another direction, we show that the simplicial volume of any mapping torus vanishes only in dimensions two and four.

Part of this talk is based on joint work with Michelle Bucher.

Orbit growth rate for maximal representations

Beatrice Pozzetti

One of the important applications of the theory of bounded cohomology is to get notions of volume for suitable representations of discrete groups in semisimple Lie groups. Maximal representations are the representations of fundamental groups of surfaces in Hermitian Lie groups that maximize one such volume. After recalling the definition and some geometric properties of maximal representations, I will discuss joint work with Andres Sambarino and Anna Wienhard in which we show that for these representations the orbit growth rate, with respect to a suitable Finsler norm on the Weyl chamber, is constant and equal to one.

Gromov norm in nonpositively curved manifolds

Shi Wang

The Gromov norm measures how efficiently a homology class can be represented by linear combinations of simplices. In this talk, we give an overview of some recent work on showing positivity of the Gromov norm. We investigate the relations between this topological norm with its underlying geometry and show how the curvature information can affect the norm. In particular, we give a positive answer to a conjecture of Gromov for a large class of manifolds.