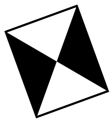
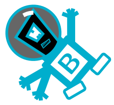
Applied Algebraic Topology: Exercises

Prof. Dr. C. Löh/M. Uschold Sheet 15, February 10, 2023

Commander Blorx currently resides on planet Apalto, recovering from past adventures and structure theorems.





Problem 1 (the hourglass)**.** While reading "Foundation", a note drops from the book into Blorx's hands. The note smells of time travel and clearly says:

Of all the timeless objects,
⧖ is most simplicial and complex.
From the following instructions,
just make homology deductions:

$$\{\emptyset, \{0\}, \dots, \{4\}, \{0,1\} \dots, \{0,4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{4,1\}, \{0,1,2\}, \{0,3,4\}\}$$

Which of the following homology deductions are valid?

$H_0(\lvert⧖\rvert) \cong_{\mathbb{Z}} \mathbb{Z}^2$	$(1, -1)$
$H_1(\lvert⧖\rvert) \cong_{\mathbb{Z}} \mathbb{Z}^2$	$(3.625, 1.875)$
$H_2(\lvert⧖\rvert) \cong_{\mathbb{Z}} \mathbb{Z}^2$	$(-3, 1.25)$

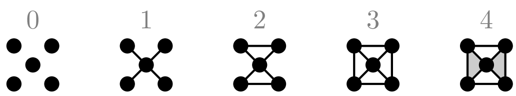
Problem 2 (to move or not to move?)**.** Blorx is tempted to follow up on the note – but also indulges the benefits of laziness. Thus, before taking further steps, he develops a theory of laziness: A simplicial complex X is *lazy* if every continuous map $|X| \longrightarrow |X|$ has a fixed point. Which of the following simplicial complexes are lazy?

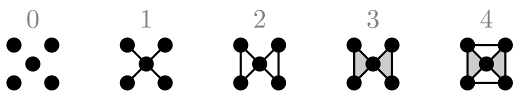
$\Delta(0)$	$(2.3, 0.45)$
$S(0)$	$(4.5, 0)$
$S(1)$	$(-2, 1)$
$\Delta(2023)$	$(6.1.3)$
$\{\emptyset, 0, 1, 2, \{0,1\}, \{0,2\}\}$	$(1.75, 2.25)$

Problem 3 (structure)**.** After completing the theory and practice of laziness, Blorx cannot continue to resist to get hold of the hourglass ⧖. He reads the complete library of Apalto; at least the barcodes. As a service for future readers, he translates the barcodes:

$(2,0)$	0.4	$\Sigma^0 \mathbb{Q}[T]$	3
$(0,\infty)$	3.5	$\Sigma^2 \mathbb{Q}[T]/(T^2)$	1
$(2,1)$	0.6	$\Sigma^2 \mathbb{Q}[T]/(T)$	0

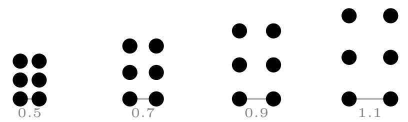
Problem 4 (manufacturing the hourglass)**.** Finally! The book describing how to manufacture the hourglass ⧖ has the barcode $(2,\infty), (2,\infty), (3,0), (3,0)$. For physics reasons, only one of the following processes is possible. Which one is it?

$(5.5, 1.5)$ 

$(5, 2.875)$ 

Please turn over

Problem 5 (the Blorx molecule)**.** To power the hourglass, Hallam's electron pump or the Blorx molecule is needed. Since Blorx is fresh out of tungsten, he synthesises the Blorx molecule. For which of the following finite subsets X of $\mathbb{R}^2$ is $|R_1(X, d_2)|$ homotopy equivalent to B ?



$(3.2, 4)$ $(2, 3.33)$ $(6.7, 3)$ $(7, 0)$

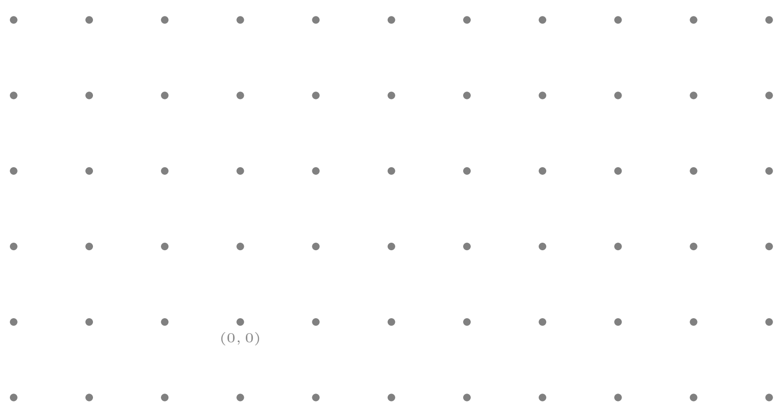
Problem 6 (making the hourglass ready for export)**.** In order to transport the hourglass ⧖, Blorx applies a simplicial map $f\colon ⧖ \longrightarrow ⧖$. What are possible outcomes for $H_1(|f|)$?

$0 \cdot \mathrm{id}_{H_1(⧖)}$ $(-2, 3)$
$1 \cdot \mathrm{id}_{H_1(⧖)}$ $(5, 1)$
$2 \cdot \mathrm{id}_{H_1(⧖)}$ $(4.3, 2.3)$
$-1 \cdot \mathrm{id}_{H_1(⧖)}$ $(-1, 1.5)$

Problem 7 (Blorx evolution)**.** The superior intellectual powers and the remarkable moral compass of Blorx can be explained by his genome BLORX. Which of the following sequences could be common ancestors of BLORX, BLOBR, and ROXOR in tree-only evolution?

BLORB $(-1, -1)$
BLORR $(6.5, -0.2)$
RLORR $(4, 0.8)$
ROORR $(-0.125, 2.625)$
LOLOL $(5, -1.125)$

Problem 8 (escape!)**.** Encouraged by his moral compass, Blorx jumps into the Sea of Tranquility and decides to leave Apalto as soon as possible with the priceless artefact. Connect the dots using brand-new raster image processors of type 2.023 and help Blorx to escape Apalto with a suitable vehicle!



No submission