Applied Algebraic Topology: Exercises

Prof. Dr. C. Löh/M. Uschold — Sheet 4, November 11, 2022

Quick check A (Čech triangle). Let $X \subset \mathbb{R}^2$ be the vertices of a regular Euclidean triangle of side length 1. Compute and illustrate $\check{C}_\varepsilon(X, \mathbb{R}^2, d_2)$ for all $\varepsilon \in \mathbb{R}_{>0}$.

Quick check B (a non-triangulable space). Show that $\{1/n \mid n \in \mathbb{N}_{>0}\} \cup \{0\}$ is *not* triangulable (with respect to the subspace topology of $\mathbb{R}$).

Quick check C (finiteness leads to compactness). Let X be a finite simplicial complex. Show that $|X|$ is compact.

Exercise 1 (open simplices? 3 credits). Is the following statement true? Justify your answer with a suitable proof or counterexample.

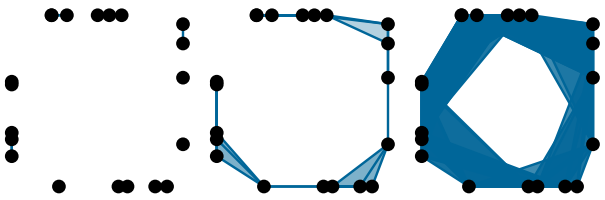
> If X is a simplicial complex and $\sigma \in X$, then $\{\xi \in |\sigma| \mid \forall_{x \in \sigma} \quad \xi_x > 0\}$ is open in $|X|$.

Exercise 2 (simplices via open stars; 3 credits). Let X be a simplicial complex and let $\sigma \subset V(X)$ be a finite non-empty subset. Show that σ is a simplex of X if and only if $\bigcap_{x \in \sigma} \operatorname{star}^\circ_X x \neq \emptyset$.

Exercise 3 (Rips complexes are flag; 3 credits). Look up the notion of *flag* simplicial complexes and prove that Rips complexes are flag.
Hints. Add the definition of "flag" that you use and add a reference for it.

Exercise 4 (small data; 3 credits). Pick four locations in Regensburg, two north of the Danube and two south of the Danube. Compute the Rips complexes of these four points with respect to the metrics "shortest way by car" and "shortest way on foot" for all radii. Add documentation/maps for your distance calculations.

Bonus problem (random Rips complexes; 3 credits). Write a program that generates pictures of Rips complexes of 20 random (uniformly distributed) points on the boundary of the square $[-1, 1] \times [-1, 1]$ with respect to the Euclidean metric. Display five samples, each with the radii 0.3, 0.9, 1.8. Explain your approach to the program.



Submission before November 18, 2022, 8:30, via GRIPS (in English or German)
The Quick checks are not to be submitted and will not be graded; they will be solved and discussed in the exercise class on November 17, 2022.