Differential Geometry I: Week 1

Prof. Dr. C. Löh/AG Ammann November 4, 2020

Reading assignment (for the lecture on November 4)**.**

- Register on GRIPS for this course.

In the first lecture, we will get acquainted with the video conferencing tool, we will discuss organisational matters, and I will give a brief overview of this course.

After this first lecture, the reading material for the second meeting will be made available. In subsequent weeks with remote teaching, I will always try to provide the material for the whole week $n+1$ on the Thursday of week n.

Reading assignment (for the lecture on November 5)**.**

- Think about how you can efficiently and successfully organise self-study: for example, when to work on the reading assignments, how to make sure that you do not just browse quickly through the material but get immersed into the subject and spend enough time on it, when/how to "meet" with fellow students to discuss the topics, which notes/annotations to take during interactive sessions, how to organise questions that arise while reading (and when/where to ask them), which additional sources to consult, how to integrate working on the exercise sheets, how to use your hardware setup in an effective way, how to schedule breaks from working at the computer screen, ...
- Register for the text chat and start interacting with other participants!
- Read Chapter 1.1.2 *Smooth manifolds.*
- Read Chapter 1.1.3 *The category of smooth manifolds.*
- Read Chapter 1.1.4 *Tangent spaces.*

This is not a lot of new material, but you will only have one day to prepare ... From week 2 on, there will be more Mathematics!

Étude (differentials)**.** Let M be a smooth manifold and $x \in M$. Compute $\mathrm{d}_x f$ for the following choices of f:

1. a constant map $M \longrightarrow M$
2. the projection $M \times M \longrightarrow M$ onto the first factor
3. the diagonal map $M \longrightarrow M \times M$
4. the inclusion $M \longrightarrow M \times \mathbb{R}$ into $M \times \{0\}$

Hints. Solutions to *Études* are not be submitted and will not be graded.

Exercises (for the session on November 9/10)**.** In this first exercise session, some basics on smooth manifolds will be discussed (e.g., as in the following exercises).

Please turn over

Exercise 0.1 (compact manifolds)**.** Let M be a smooth manifold. Which of the following statements are true? Justify your answer with a suitable proof or counterexample.

1. If M is compact, then M admits a finite smooth atlas.
2. If M admits a finite smooth atlas, then M is compact.

Exercise 0.2 (smooth atlas on $\mathbb{R}$)**.** We equip $\mathbb{R}$ with the standard topology and consider the map

$$\begin{aligned} \varphi\colon \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \begin{cases} x & \text{if } x \leq 0, \\ 2 \cdot x & \text{if } x > 0. \end{cases} \end{aligned}$$

Which of the following statements are true? Justify your answer with a suitable proof or counterexample.

1. The set $\{\varphi\}$ is a smooth atlas for $\mathbb{R}$.
2. The set $\{\varphi, \mathrm{id}_{\mathbb{R}}\}$ is a smooth atlas for $\mathbb{R}$.

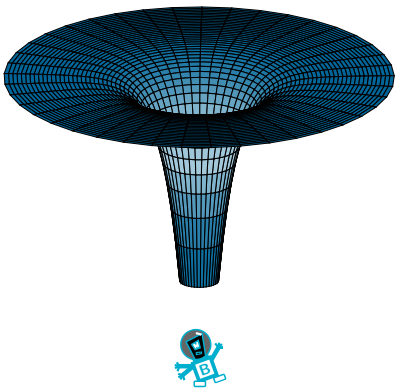
Exercise 0.3 (local coordinates)**.** Let $\alpha \in \mathbb{R}$ and let $f\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the rotation around 0 about the angle α.

1. Express f in local coordinates with respect to the smooth chart $\mathrm{id}_{\mathbb{R}^2}$.
2. Express f in local coordinates with respect to polar coordinates.

Exercise 0.4 (differentials via derivations)**.** Let $f\colon M \longrightarrow N$ be a smooth map between smooth manifolds and let $x \in M$.

1. Describe the differential $\mathrm{d}_x f\colon \mathrm{T}_x M \longrightarrow \mathrm{T}_x N$ in terms of derivations.
2. Prove that this description coincides with the description of $\mathrm{d}_x f$ in terms of curves.

Exercise 0.5 (the real world)**.** Give examples of "real-world" situations that can be modelled by smooth manifolds.



No submission! Questions of this type will be discussed in the first exercise session on November 9/10.