Group Cohomology – Etudes

Prof. Dr. C. Löh/D. Fauser/J. P. Quintanilha/J. Witzig Sheet 0, April 25, 2019

Exercise 1 (group rings)**.** Let $G := \mathbb{Z}/8$ and let $t := [1] \in \mathbb{Z}/8$.

1. Is $\mathbb{Z}G$ a commutative ring?
2. Compute $(t-1) \cdot \sum_{j=0}^{7} t^j$ in $\mathbb{Z}G$.
3. Is $\mathbb{Z}G$ isomorphic to $\mathbb{Z}[e^{2\pi i/8}] \subset \mathbb{C}$?
4. Is $t^4 - 1$ a unit of $\mathbb{Z}G$?

Exercise 2 (invariants)**.** Let G be a group. For each of the following $\mathbb{Z}G$-modules, compute the invariants.

1. $\mathbb{R}$ with the trivial G-action
2. $\operatorname{map}(G, \mathbb{R})$, the space of $\mathbb{R}$-valued functions on G, with respect to the G-action by right translation on G
3. $\ell^2(G, \mathbb{C})$, the space of square-summable complex functions on G, with respect to the G-action by right translation on G
4. $A \times B$, where A and B are $\mathbb{Z}G$-modules, with respect to the diagonal G-action on $A \times B$.

Exercise 3 (basic homological algebra)**.**

1. What is the definition of *chain complexes* and *chain maps*?
2. What are typical examples?
3. What is the *homology* of a chain complex?
4. How can homology be computed?
5. How does all this relate to exactness?
6. What is *homotopy* invariance in homological algebra?
7. Why did we introduce chain complexes in Commutative Algebra or Algebraic Topology or Any-Other-Course?

Hints. In case you don't know any homological algebra: Don't panic! Basic notions from homological algebra will also be quickly reviewed in the lectures.

Exercise 4 (basic category theory)**.**

1. What is the definition of a *category*?
2. What is the definition of a *functor* between categories?
3. Give examples of categories and functors between them. (How) Did these arise naturally in previous courses?

Hints. In case you don't know any category theory: Don't panic! Categories and functors will also be quickly reviewed in the lectures.

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no submission!

These problems will be discussed in the exercise classes in the second week.