Group Cohomology – Etudes

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Exercise 1 (group (co)homology in degree 0)**.** Let $G := \mathbb{Z}/2$. For a $\mathbb{Z}$-module Z, let Z^- be the left $\mathbb{Z}G$-module whose underlying additive group is Z and where the non-trivial element of G acts by multiplication with -1. Compute the following (co)homology groups:

1. $H^0(G; \mathbb{R}^-)$
2. $H_0(G; \mathbb{R}^-)$
3. $H^0(G; \mathbb{Z}^-)$
4. $H_0(G; \mathbb{Z}^-)$
5. $H^0(G; \mathbb{Z}/2^-)$
6. $H_0(G; \mathbb{Z}/2^-)$

Exercise 2 (simplicial chains)**.** Let $G := \mathbb{Z}$ and let $t := 1 \in \mathbb{Z} = G$. Which of the following elements of $C_1(G)$ are cycles (i.e., in the kernel of ∂_1)?

1. $2019 \cdot (t^{2019}, t^{2019})$
2. $2019 \cdot (t, t^{2019})$
3. $1 \cdot (t^0, t) + 1 \cdot (t, t^2) - 1 \cdot (t^0, t^2)$

Exercise 3 (free groups)**.** Let F be the free group of rank 2, freely generated by $\{a, b\}$ (Appendix A.1). Which of the following equalities hold in F ?

1. $(aba^{-1})^{2019} = ab^{2019}a^{-1}$
2. $aba^{-1} \cdot ab^2a^{-1} = b^3$
3. $[a, b]^2 = [a^2, b^2]$
4. $[a^2, a^{-1}b^2] = ab^2a^{-2}b^{-2}a$

Exercise 4 (summary)**.** Write a summary of Chapter 1.1 (Foundations: The group ring) and Chapter 1.2 (The basic definition of group (co)homology), keeping the following questions in mind:

1. How can one work with the group ring? What are basic examples?
2. What are important examples and constructions of $\mathbb{Z}G$-modules?
3. What are the domain categories for group (co)homology?
4. How are the simplicial/bar resolutions constructed? Why?
5. How is group (co)homology defined in terms of the simplicial resolution?
6. Did you check all the little things that we did not discuss in detail in the lectures?

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no submission!