

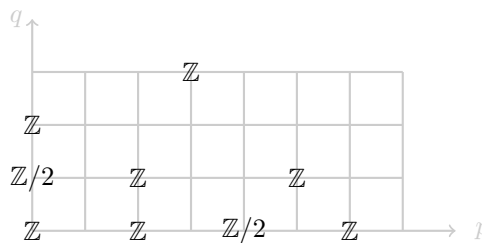
# Group Cohomology – Etudes

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**Exercise 1** (spectral sequence). The following diagram is the second page of a homological spectral sequence (of  $\mathbb{Z}$ -modules; all unlabelled modules are trivial).

1. Which components of the second differential have to be trivial? Which could be non-trivial? What about the third differential? What about all differentials?
2. What does this imply for the  $\infty$ -page?



**Exercise 2** (five lemma).

1. View the situation of the five lemma as a double complex.
2. Spell out the double complex spectral sequences of this double complex (i.e., what are the first pages? To what do they converge?, ...).
3. Use the double complex spectral sequences to prove the five lemma.

**Exercise 3** (long exact homology sequence).

1. Spell out the filtration spectral sequence for the inclusion of a single sub-complex of a chain complex (i.e., what is the first page? To what does it converge?, ...).
2. Use the filtration spectral sequence in this situation to establish the long exact homology sequence associated with a short exact sequence of chain complexes.

**Exercise 4** (summary). Write a summary of Chapter 3.1 (Derived functors), keeping the following questions in mind:

1. Why does one need derived functors?
2. What are derived functors?
3. How can derived functors be constructed/computed?
4. How do derived functors relate to group (co)homology?
5. What is the derived category and what are total derived functors?
6. Did you check all the little things that we did not discuss in details in the lectures?

no submission!