Group Cohomology – Etudes

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Exercise 1 (classifying spaces for $\mathbb{Z}$). Which of the following spaces are classifying spaces of $\mathbb{Z}$ (when equipped with a suitable CW-structure, etc.)?

1. $S^1 \times \mathbb{R}^{2019}$
2. $\mathbb{R}^2 \setminus \{0\}$
3. $\mathbb{R}^2 \setminus \{0, (1, 0)\}$
4. $\mathbb{R}^{2019} \setminus \{0\}$
5. $S^1 \vee D^{2019}$
6. $S^1 \times S^{2019}$

Exercise 2 (two-dimensional classifying spaces). Which of the following groups admit a classifying space of dimension 2 ? Here, Γ_g denotes the surface group of genus g and F_n denotes the free group of rank n.

1. $\Gamma_{2018} * \Gamma_{2019} * F_{2019}$
2. $\Gamma_{2018} \times \Gamma_{2019}$
3. $\mathbb{Z}^{2019}$
4. $\mathbb{Z}/2019 * \Gamma_{2019}$
5. $\Gamma_{2018} * (F_{2019} \times F_{2018})$
6. $\mathbb{Z}$

Exercise 3 (presentation complexes). Draw the presentation complexes of the following group presentations. Do you recognise the groups?

1. $\langle x \mid x^2 \rangle$
2. $\langle x, y \mid x \rangle$
3. $\langle x, y \mid [x, y] \rangle$
4. $\langle x, y \mid x^2, y^2 \rangle$

Exercise 4 (summary). Write a summary of Chapter 4.1 (Classifying spaces) keeping the following questions in mind:

1. What is the definition of classifying spaces for groups?
2. What is an example of a functorial construction?
3. What are typical “nice” classifying spaces?
4. Why are classifying spaces useful in group (co)homology?
5. How do classifying spaces compare to projective resolutions?
6. Did you check all the little things that we did not discuss in detail in the lectures?

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no submission!