

Group Cohomology – Etudes

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Exercise 1 (cohomological dimension). Determine the cohomological dimension of the following groups.

1. $\mathbb{Z}/2019 \times F_{2019}$
2. $\mathrm{GL}_{2019}(\mathbb{F}_{2017})$
3. $\mathrm{GL}_{2019}(\mathbb{Z})$
4. $\Gamma_{2017} * \Gamma_{2018} * \Gamma_{2019}$
5. $\Gamma_{2017} \times \Gamma_{2018} \times \Gamma_{2019}$
6. $\mathbb{Z}^{2019} \times F_{2020}$

Exercise 2 (sphere actions). Which of the following groups admit a free action on some sphere?

$$\mathbb{Z}/2019, \quad \mathbb{Z}/4 \times \mathbb{Z}/9, \quad \mathrm{GL}_{2019}(\mathbb{F}_{2017}), \quad \mathbb{Z}$$

Exercise 3 (summary). Write a summary of Chapter 4.2 (Finiteness conditions) and Chapter 4.3 (Application: Free actions on spheres), keeping the following questions in mind:

1. Which finiteness conditions for groups do you know?
2. How are they related?
3. What are typical examples?
4. How can they be computed?
5. Which necessary conditions do you know for finite groups to admit a free action on a sphere? How does this relate to group (co)homology?
6. How does this work in concrete examples?
7. Did you check all the little things that we did not discuss in detail in the lectures?

Exercise 4 (exam). Due to a tragic malfunction in space-time you end up as examiner in an oral exam on Group Cohomology.

1. Which questions will you ask on basic notions?
2. Which questions will you ask on applications?
3. Which questions will you ask on the different views and methods of computation on group (co)homology?
4. Which examples will you discuss during the exam?

Try out your questions on your fellow students!

no submission!