

Group Cohomology – Etudes

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Exercise 1 (cohomology in degree 1). Compute the following cohomology groups (with respect to the trivial action on the coefficients).

1. $H^1(\mathbb{Z}; \mathbb{Z}/2)$
2. $H^1(\mathbb{Z}/2; \mathbb{Z}/3)$
3. $H^1(S_{2019}; \mathbb{Z}/2019)$
4. $H^1(S_{2020}; \mathbb{Z}/2020)$

Exercise 2 (presentations). Which groups are described by the following presentations? Use universal properties to verify your claims.

1. $\langle a, b \mid a \rangle$
2. $\langle a, b \mid ab \rangle$
3. $\langle a, b \mid ab^2 \rangle$
4. $\langle a, b \mid aba^{-1} \rangle$
5. $\langle a, b \mid a^3, b^2, aba^{-1}b^{-1} \rangle$
6. $\langle a, b \mid a^3, b^2, aba^{-2} \rangle$

Exercise 3 (extensions). Do there exist extensions of the following types?

1. $0 \longrightarrow \mathbb{Z}/2 \longrightarrow \mathbb{Z}/6 \longrightarrow \mathbb{Z}/3 \longrightarrow 0$
2. $0 \longrightarrow \mathbb{Z}/2 \longrightarrow S_3 \longrightarrow \mathbb{Z}/3 \longrightarrow 0$
3. $0 \longrightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \longrightarrow 0$
4. $0 \longrightarrow \mathbb{Z}/2 \longrightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \longrightarrow 1$

Exercise 4 (summary). Write a summary of Chapter 1.3 (Degree 0: (Co)Invariants), Chapter 1.4 (Degree 1: Abelianisation and homomorphisms), and Chapter 1.5 (Degree 2: Presentations and extensions) keeping the following questions in mind:

1. How can one compute group (co)homology in low degrees?
2. What are typical examples?
3. What are typical applications?
4. What kind of finiteness obstructions can we get from group homology in low degrees?
5. Did you check all the little things that we did not discuss in detail in the lectures?

no submission!