

Group Cohomology – Etudes

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Exercise 1 (projectivity). Which of the following modules are projective?

1. $\mathbb{Z}/2019$ over \mathbb{Z} ?
2. \mathbb{Q} over \mathbb{Z} ?
3. $\prod_{\mathbb{N}} \mathbb{Q}$ over \mathbb{Q} ?
4. $\mathbb{Z} \times \{0\} \subset \mathbb{Z} \times \mathbb{Z}$ over $\mathbb{Z} \times \mathbb{Z}$?

Exercise 2 (homology of cyclic groups). Compute the following (co)homology groups (where $\mathbb{Z}/2019$ acts trivially on the coefficients):

1. $H_*(\mathbb{Z}/2019; \mathbb{Z})$
2. $H_*(\mathbb{Z}/2019; \mathbb{Z}/2019)$
3. $H_*(\mathbb{Z}/2019; \mathbb{Q})$
4. $H_*(\mathbb{Z}/3; \mathbb{Z}/2019)$

Exercise 3 (p -groups). Let $p \in \mathbb{N}$ be a prime.

1. How can one prove that p -groups are solvable?
2. Give examples of p -groups that do *not* contain a cyclic subgroup of index p .

Exercise 4 (example list). Start a list of groups and their (co)homology, containing the following data (as far as you know it):

- name of the group
- standard notation of the group
- standard presentations of the group
- geometric relevance of the group
- homology of the group (at least with \mathbb{Z} -coefficients) and method of computation
- cohomology of the group (at least with \mathbb{Z} -coefficients) and method of computation
- applications of this group (co)homology

Update this list during this course (don't forget to add the examples from the exercises!).

no submission!