Group Cohomology – Etudes

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Exercise 1 (amenability). Which of the following groups are amenable?

1. $\mathbb{Z} \times \mathbb{Z}/2019$

2. D_∞

3. the Heisenberg group

4. the symmetric group $\mathrm{Sym}(\mathbb{N})$ of all bijections $\mathbb{N} \longrightarrow \mathbb{N}$

5. $\mathrm{SL}_2(\mathbb{Z})$

6. $\prod_{\mathbb{N}} \mathbb{R}$

Exercise 2 (uniformly finite chains). Which of the following "sums" describe uniformly finite chains in $C_1^{\mathrm{uf}}(\mathbb{Z}; \mathbb{Z})$? Draw them! If they describe uniformly finite chains: What is their boundary?

1. $\sum_{n \in \mathbb{Z}} 1 \cdot (n, n+1)$

2. $\sum_{n \in \mathbb{N}} 1 \cdot (n, n+1)$

3. $\sum_{n \in \mathbb{N}} 1 \cdot (1, n)$

4. $\sum_{n \in \mathbb{N}} 1 \cdot (-n, n)$

5. $\sum_{n \in \mathbb{N}} n \cdot (n, n+1)$

6. $\sum_{n \in \mathbb{N}} 2019 \cdot (n, n+2019)$

Exercise 3 (uniformly finite homology of finite groups). Let G be a finite group. Compute $H_*^{\mathrm{uf}}(G; \mathbb{Z})$ in as many ways as you can!

Exercise 4 (summary). Write a summary of Chapter 2.1 (Foundations: Geometric group theory), keeping the following questions in mind:

1. What is the (geo)metric setup in geometric group theory?

2. What are typical examples?

3. What is amenability?

4. Did you check all the little things that we did not discuss in detail in the lectures?