

Algebraic Topology I – Exercises

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Sheet –1, October 13, 2015

Exercise 1 (continuous maps and topological properties). Let P be one of the following properties of (subsets of) topological spaces:

open, closed, compact, connected, path-connected, Hausdorff

Answer the following questions (and prove that your answer is correct):

1. How is property P defined?
2. Let X and Y be topological spaces, let $f: X \rightarrow Y$ be continuous, and let $A \subset X$ have property P . Does then also $f(A)$ have property P ?
3. Let X and Y be topological spaces, let $f: X \rightarrow Y$ be continuous, and let $B \subset Y$ have property P . Does then also $f^{-1}(B)$ have property P ?

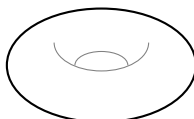
Exercise 2 (homeomorphisms). Let X and Y be topological spaces and let $f: X \rightarrow Y$ be a bijective continuous map.

1. Give an example that shows that f in this situation in general does not need to be a homeomorphism.
2. Provide reasonable assumptions on X and Y that guarantee that f in this situation will be a homeomorphism.

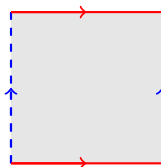
Exercise 3 (universal property of the product topology).

1. What is the universal property of the product topology?
2. Prove that the product topology satisfies this universal property.
3. What is the universal property of product groups, product measurable spaces, ... ?

Exercise 4 (torus via glueing). The 2-torus is defined as $T := S^1 \times S^1$, where we endow $S^1 \subset \mathbb{R}^2$ with the subspace topology and T with the product topology.



1. How can one describe the glueing of the unit square as indicated in the sketch below via the quotient topology?



2. Let Q be this quotient space. Prove in detail that T and Q are homeomorphic.
3. How is this related to the classical Asteroids game by Atari?
<http://www.atari.com/arcade/arcade/asteroids-online#!/arcade/asteroids/play>
4. Are T and S^1 homeomorphic? Prove that your answer is correct.

Please turn over

Bonus Problem (homeomorphism groups). Let G be a group. Prove that there exists a topological space whose homeomorphism group is isomorphic to G .

Hints. The *homeomorphism group* of a topological space X is the set of all homeomorphisms $X \rightarrow X$ with the group structure given by composition of maps.

no submission;
these exercises will be discussed in the exercise class on October 14, 2015