Algebraic Topology – Etudes

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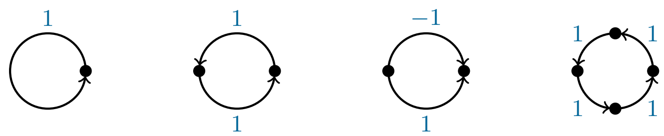
Exercise 1 (singular chains on the torus). We consider the usual description of the torus T as quotient of the unit square $[0,1]\times[0,1]$ (Figure 1.6). In the unit square, we use the following notation for singular 2-simplices: If $v_0, v_1, v_2 \in [0,1]\times[0,1]$, then we consider the associated affine linear 2-simplex

$$\begin{aligned}[v_0, v_1, v_2] \colon \Delta^2 &\longrightarrow [0,1]\times[0,1] \\ (t_0, t_1, t_2) &\longmapsto t_0 \cdot v_0 + t_1 \cdot v_1 + t_2 \cdot v_2.\end{aligned}$$

Which of the following singular chains are cycles in $C_2([0,1]\times[0,1];\mathbb{Z})$? Which of them describe cycles in $C_2(T;\mathbb{Z})$? In $C_2(T;\mathbb{Z}/2)$? Illustrate!

1. $1 \cdot [(0,0),(0,1),(1,1)]$
2. $1 \cdot [(0,0),(0,1),(1,1)] + 1 \cdot [(0,0),(1,0),(1,1)]$
3. $1 \cdot [(0,0),(0,1),(1,1)] - 1 \cdot [(0,0),(1,0),(1,1)]$
4. $1 \cdot [(0,0),(0,1),(1,1)] + 1 \cdot [(0,0),(1,1),(1,0)]$

Exercise 2 (singular homology classes on the circle). Show that the following singular 1-cycles on S^1 all represent the same class in $H_1(S^1;\mathbb{Z})$ (where all the paths are parametrised at constant speed).



Exercise 3 (homology of chain complexes). Compute the homology of the following chain complexes of $\mathbb{Z}$-modules:

1. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{0} \mathbb{Z}^2 \xrightarrow{0} \mathbb{Z} \longrightarrow 0 \longrightarrow \cdots$
2. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \longrightarrow 0 \longrightarrow \cdots$
3. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Q} \xrightarrow{2} \mathbb{Q} \xrightarrow{0} \mathbb{Q} \longrightarrow 0 \longrightarrow \cdots$
4. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Z}/2019 \xrightarrow{3} \mathbb{Z}/2019 \xrightarrow{673} \mathbb{Z}/2019 \longrightarrow 0 \longrightarrow \cdots$

Exercise 4 (summary). Write a summary of Chapter 4.1 (Construction), keeping the following questions in mind:

1. What is the geometric idea behind singular homology?
2. Which algebraic objects are used to implement this geometric idea?
3. How can one manipulate singular chains/cycles/boundaries?

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no submission!