

Algebraic Topology – Etudes

Prof. Dr. C. Löh/D. Fauser/J. Witzig

Sheet 14, January 31, 2019

Exercise 1 (CW-complexes). Are the following filtrations CW-structures on the unit interval $[0, 1]$?

1. $\emptyset \subset \{0\} \subset [0, 1]$
2. $\emptyset \subset \{0, 1/2, 1\} \subset [0, 1]$
3. $\emptyset \subset [0, 1/2] \subset [0, 1]$
4. $\emptyset \subset [0, 1) \subset [0, 1]$
5. $\emptyset \subset [0, 1] \setminus \{1/2\} \subset [0, 1]$
6. $\emptyset \subset \{1/n \mid n \in \mathbb{N}_{>0}\} \subset [0, 1]$

Exercise 2 (cellular maps). For each of the following maps $S^1 \rightarrow S^1$ find a CW-structure on S^1 such that the given map is cellular and find one such that the given map is *not* cellular.

1. $[t] \mapsto [2019 \cdot t \bmod 1]$
2. $[t] \mapsto [t + 1/2019 \bmod 1]$
3. $[t] \mapsto [t^{2019} \bmod 1]$
4. $[t] \mapsto [\sin(\pi \cdot t) \bmod 1]$

Exercise 3 (cellular homology). Choose two different CW-structures on $S^1 \times S^1$. In the following, we will consider cellular chain complexes and cellular homology with respect to singular homology with \mathbb{Z} -coefficients.

1. Compute the corresponding cellular chain complexes explicitly.
2. Compute the corresponding cellular homology.

Exercise 4 (summary). Write a summary of Chapter 5.1 (The Category of CW-Complexes) and Chapter 5.2 (Cellular Homology; 5.2.1–5.2.2), keeping the following questions in mind:

1. What are typical examples of CW-complexes and cellular maps?
2. What is the geometric idea of cellular homology?
3. What is the actual construction of cellular homology?
4. How can cellular homology be computed?

no submission!