

Algebraic Topology – Etudes

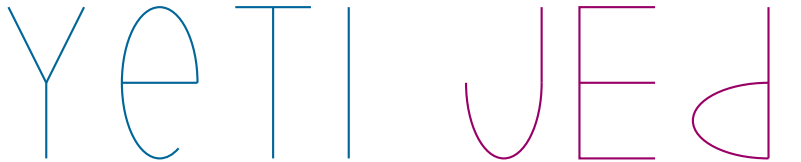
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Sheet 15, February 7, 2019

Exercise 1 (Euler characteristic). Show that the following spaces admit a finite CW-structure and compute their Euler characteristic:

1. $S^{2018} \times S^{2019}$
2. $(S^1)^{2019}$
3. $\bigvee^{2019} S^1$
4. $\mathbb{R}P^{2018} \times \mathbb{R}P^{2019}$
5. $\mathbb{R}P^{2019} \times \mathbb{R}P^{2019}$

Exercise 2 (Yeti vs. Jedi). We consider the following two subspaces of \mathbb{R}^2 :



1. Are these spaces homeomorphic?
2. Which connected components of YeTI are homeomorphic to which connected components of JEdI?
3. Are YeTI and JEdI homotopy equivalent?
4. Compute all homotopy groups of all connected components.
5. Compute $H_n(\cdot; \mathbb{Z})$ of these spaces for all $n \in \mathbb{Z}$.
6. Compute the Euler characteristic of these spaces.
7. Which connected components admit a 2019-sheeted connected covering?

Exercise 3 (summary). Write a summary of Chapter 5.2 (Cellular Homology; 5.2.3) and Chapter 5.3 (The Euler Characteristic) keeping the following questions in mind:

1. How can homology theories on CW-complexes be compared?
2. What consequences does this have for practical computations?
3. What is the Euler characteristic?
4. How can the Euler characteristic be computed?
5. What are typical applications of the Euler characteristic?

Exercise 4 (summary summary). Read all your summaries of all chapters! In hindsight, would you change them?

no submission!