

Algebraic Topology – Etudes

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Exercise 1 (functors on pairs of spaces). Give reasonable definitions of functors $\mathbf{Top}^2 \rightarrow \mathbf{Top}$ that deserve the following names:

1. subspace functor
2. ambient space functor
3. quotient space functor

Exercise 2 (morphisms in the simplex category). In the simplex category Δ , describe the following sets of morphisms explicitly and draw schematic pictures of these morphisms:

1. $\text{Mor}_\Delta(\Delta(1), \Delta(2))$
2. $\text{Mor}_\Delta(\Delta(0), \Delta(2))$
3. $\text{Mor}_\Delta(\Delta(2), \Delta(1))$
4. $\text{Mor}_\Delta(\Delta(2), \Delta(2))$

Exercise 3 (homotopy). Describe for the following types of maps all possible homotopy classes:

1. $S^0 \rightarrow [0, 1]$
2. $S^0 \rightarrow S^0$
3. $[0, 1] \rightarrow S^0$
4. $\mathbb{R}^2 \rightarrow \mathbb{R}^2$

Exercise 4 (summary). Write a summary of Chapter 1.2 (Categories and Functors), keeping the following questions in mind:

1. Which examples of categories/functors did you encounter in other courses?
2. Why are categories/functors important in Algebraic Topology?
3. What properties do representable functors have?

no submission!