Algebraic Topology – Etudes

Prof. Dr. C. Löh/D. Fauser/J. Witzig Sheet 3, November 1, 2018

Exercise 1 (homotopy). Let X be a topological space and let $f \in \operatorname{map}(f, f)$. Prove or disprove:

1. If $f \simeq \mathrm{id}_X$, then f is bijective.
2. If $f \simeq \mathrm{id}_X$, then $f \circ f \simeq f$.
3. If $f \circ f \simeq f$, then $f \simeq \mathrm{id}_X$.
4. If $f^{2018} \simeq \mathrm{id}_X$, then f is a homotopy equivalence.

Exercise 2 (homotopy invariant functors). Let $F\colon \mathsf{Top} \longrightarrow \mathsf{Ab}$ be a homotopy invariant functor. Prove or disprove:

1. $F(D^{2018}) \cong_{\mathsf{Ab}} F(\mathbb{R}^{2019})$.
2. $F(S^{2018}) \not\cong_{\mathsf{Ab}} \mathbb{Z}^{2018}$.
3. $F(\mathbb{R}P^{2018}) \cong_{\mathsf{Ab}} F(\mathbb{R} \times \mathbb{R}P^{2018})$.
4. If X is contractible, then $F(X) \cong_{\mathsf{Ab}} \{0\}$.
5. If $F(\mathbb{R}P^{2018}) \cong_{\mathsf{Ab}} \mathbb{Z}$, then $F(S^{2018}) \not\cong_{\mathsf{Ab}} \mathbb{Z}$.

Exercise 3 (classification problem). In this exercise, you may assume that the theorem on existence of "interesting" homotopy invariant functors holds. Classify the following spaces up to homeomorphism/homotopy equivalence.

1. $\mathbb{R}^{2018}$
2. $\mathbb{R}^{2019}$
3. S^{2018}
4. D^{2019}
5. $S^0 \times S^{2019}$

Exercise 4 (summary). Write a summary of Chapter 1.3 (Homotopy and Homotopy Invariance), keeping the following questions in mind:

1. What is homotopy/homotopy equivalence?
2. What are basic examples?
3. What is homotopy invariance?
4. How can homotopy invariance be used?

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no submission!