

# Algebraic Topology – Etudes

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**Exercise 1** (lifts of loops). We consider the following covering maps

$$\begin{aligned} p: \mathbb{R} &\longrightarrow S^1 \\ t &\longmapsto [t \bmod 1] \\ q: S^1 &\longrightarrow S^1 \\ t &\longmapsto [2 \cdot t \bmod 1] \\ r: S^1 &\longrightarrow S^1 \\ t &\longmapsto [3 \cdot t \bmod 1] \end{aligned}$$

and the (pointed) loop

$$\begin{aligned} \gamma: (S^1, e_1) &\longrightarrow (S^1, e_1) \\ [t] &\longmapsto [t]. \end{aligned}$$

Which of the following *loops* admit a  $p$ -lift, a  $q$ -lift, or an  $r$ -lift?

$$\gamma, \quad \gamma * \bar{\gamma}, \quad \gamma * \gamma, \quad \gamma^{*2017}, \quad \gamma^{*2018}, \quad \gamma^{*2019}$$

**Exercise 2** (deck transformations). Give examples of non-trivial deck transformations of the following covering maps!

1.  $\mathbb{R} \longrightarrow S^1, t \longmapsto [t \bmod 1]$
2.  $S^1 \longrightarrow S^1, [t] \longmapsto [2 \cdot t \bmod 1]$
3.  $S^2 \longrightarrow \mathbb{R}P^2, x \longmapsto \{x, -x\}$

**Exercise 3** (non-coverings). Why are there no coverings of the following types?

1.  $S^1 \longrightarrow \mathbb{R}$
2.  $\mathbb{R}P^2 \longrightarrow S^2$
3.  $S^2 \longrightarrow S^1$
4.  $\mathbb{R}^2 \setminus \{-e_1, e_1\} \longrightarrow \mathbb{R}^2 \setminus \{0\}$

**Exercise 4** (SchnüffelTron3000).

1. The Blorxian Space Agency launched the satellite SchnüffelTron3000 that constantly monitors the location of all cars on the surface of Earth (i.e., their longitude and latitude, but no height information). Which information on the actual location of a car on a circular car park ramp can be derived from such information? How does this relate to covering theory?
2. How does this relate to the branches of the logarithm function in complex analysis?

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no submission!