Algebraic Topology – Etudes

Prof. Dr. C. Löh/D. Fauser/J. Witzig Sheet 8, December 6, 2018

Exercise 1 (non-trivial coverings)**.** Which of the following spaces admit non-trivial coverings?

1. $\mathbb{R}^2 \setminus \{0\}$
2. $\mathbb{R}^{2018} \setminus \{0\}$
3. $S^{2018} \setminus \{e_1, -e_1\}$
4. $\mathbb{R}^2 \setminus \{e_1, -e_1\}$

Exercise 2 ("random" coverings)**.** Let $(T, t_0) := (S^1, e_1) \times (S^1, e_1)$.

1. Roll four dice; let a, b, c, d be the results and let
$$H := \operatorname{Span}_{\mathbb{Z}} \left\{ \begin{pmatrix} a-1 \\ b-1 \end{pmatrix}, \begin{pmatrix} c-1 \\ d-1 \end{pmatrix} \right\} \subset \mathbb{Z}^2.$$
2. Choose an isomorphism $\pi_1(T, t_0) \cong_{\mathsf{Group}} \mathbb{Z}^2$ and consider the subgroup H' corresponding to H under this isomorphism.
3. Draw the path-connected, pointed, covering of (T, t_0) associated with H'.
4. Iterate!
5. What is the probability that the resulting total space is homeomorphic to $\mathbb{R}^2$?

Exercise 3 (exact sequences)**.** Refresh your memory of the following algebraic terms (Appendix A.6.1):

1. (short) exact sequence
2. split exact sequence
3. five lemma
4. flat module

Exercise 4 (summary)**.** Write a summary of Chapter 2.3 (Covering Theory) and Chapter 2.4 (Applications), keeping the following questions in mind:

1. What are important examples of (non-trivial) coverings?
2. Which lifting properties do coverings have? Why?
3. Why are coverings compatible with homotopy groups?
4. How can coverings be classified?
5. How can covering theory be used to compute fundamental groups?
6. Which applications do fundamental groups and covering theory have?

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no submission!