Algebraic Topology – Etudes

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Exercise 1 (suspension). Draw the suspensions of the following spaces and compute their (ordinary) homology:

1. $\{0, 1, 2\} \subset \mathbb{R}$
2. D^2
3. $S^1 \sqcup S^1$
4. $(S^1, e_1) \vee (S^1, e_1)$

Exercise 2 (long exact sequences). Let $\big((h_k)_{k\in\mathbb{Z}}, (\partial_k)_{k\in\mathbb{Z}}\big)$ be a homology theory on Top^2 and let (X, A) be a pair of spaces.

1. Write down the long exact sequence of this pair with respect to the homology theory $\big((h_k)_{k\in\mathbb{Z}}, (\partial_k)_{k\in\mathbb{Z}}\big)$. What can you conclude from this sequence if X is contractible? What if A is contractible?

   Apply this to $(\mathbb{R}^2, \mathbb{R}^2 \setminus S^1)$.
2. Let $B \subset A$. Write down the long exact sequence of the triple (X, A, B) with respect to $\big((h_k)_{k\in\mathbb{Z}}, (\partial_k)_{k\in\mathbb{Z}}\big)$. What can you conclude from this sequence if the inclusion $B \longrightarrow A$ is a homotopy equivalence? What if the inclusion $A \longrightarrow X$ is a homotopy equivalence? What if the inclusion $B \longrightarrow X$ is a homotopy equivalence?

   Apply this to $(\mathbb{R}^2, S^1, \{e_1\})$.

Exercise 3 (excision). Which of the following pairs of spaces are related by excision (as in the excision axiom)?

1. $(\mathbb{R}^{2018}, \{0\})$ and $(\mathbb{R}^{2018} \setminus \{0\}, \emptyset)$
2. $(\mathbb{R}^2, S^1)$ and $(\mathbb{R}^2 \setminus \{0\}, \emptyset)$
3. $(\mathbb{R}^2, \mathbb{R}^2 \setminus \{0\})$ and $([0, 1] \times [0, 1], ([0, 1] \times [0, 1]) \setminus \{0\})$
4. $(\mathbb{R}^2, \mathbb{R}^2 \setminus \{2 \cdot e_1\})$ and $(\mathbb{R}^2 \setminus D^2, \mathbb{R}^2 \setminus (D^2 \cup \{2 \cdot e_1\}))$
5. $(\mathbb{R}^2, \mathbb{R}^2 \setminus \{e_1\})$ and $(\mathbb{R}^2 \setminus D^2, \mathbb{R}^2 \setminus D^2)$
6. $(\mathbb{R}^2, \mathbb{R}^2 \setminus \{0, 2 \cdot e_1\})$ and $(D^2, D^2 \setminus \{0\})$

Exercise 4 (summary). Write a summary of Chapter 3.1 (The Eilenberg-Steenrod Axioms), keeping the following questions in mind:

1. What do the axioms mean geometrically?
2. How can the axioms be used in computations?

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no submission!