

Algebraic Topology: Études

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Exercise 1 (basic point-set topology).

1. What is the definition of a *topological space*?
2. Which inheritance properties do closed sets in a topological space have?
3. How does a metric induce a topology?
4. What is the definition of a *continuous map* between topological spaces?
5. Why is the composition of two (composable) continuous maps continuous?

Exercise 2 (T as in Topology). We consider

$$T := \{(t, 0) \mid t \in [-1, 1]\} \cup \{(0, t) \mid t \in [-1, 0]\} \subset \mathbb{R}^2,$$

endowed with the subspace topology of \mathbb{R}^2 .

1. Sketch T !
2. What is the definition of *compactness*?
3. Is the topological space T compact? Is $T \setminus \{(0, 0)\}$ compact?
4. What is the definition of *path-connectedness*?
5. Is the topological space T path-connected? Is $T \setminus \{(0, 0)\}$ path-connected?
6. Is T homeomorphic to \mathbb{R} ? Or to S^1 ?

Exercise 3 (subspaces of \mathbb{R}^3). Give explicit subspaces (as sets, in coordinates) of \mathbb{R}^3 that are homeomorphic to the following spaces:

1. the standard 3-simplex Δ^3
2. the torus $S^1 \times S^1$
3. the Möbius strip

Check your formulas with a visualisation tool!

Exercise 4 (Library). Select five books on category theory and five books on point-set topology. For both topics, compare these books:

1. Which books contain many examples?
2. Which books focus on theory?
3. In which books can you find terms/theorems quickly?
4. Which books contain useful exercises?

no submission!