Algebraic Topology: Études

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Exercise 1 (CW-complexes)**.** Are the following filtrations CW-structures on the unit interval $[0,1]$? If so, compute the cellular chain complex (with respect to $H_*(\,\cdot\,;\mathbb{Z})$), the cellular homology, and the Euler characteristic.

1. $\emptyset \subset \{0\} \subset [0,1]$
2. $\emptyset \subset \{0, 1/2, 1\} \subset [0,1]$
3. $\emptyset \subset [0, 1/2] \subset [0,1]$
4. $\emptyset \subset [0,1) \subset [0,1]$
5. $\emptyset \subset [0,1] \setminus \{1/2\} \subset [0,1]$
6. $\emptyset \subset \{1/n \mid n \in \mathbb{N}_{>0}\} \subset [0,1]$

Exercise 2 (cellular homology)**.** Choose two different CW-structures on $S^1 \times S^1$. In the following, we will consider cellular chain complexes and cellular homology with respect to singular homology with $\mathbb{Z}$-coefficients.

1. Compute the corresponding cellular chain complexes explicitly.
2. Compute the corresponding cellular homology.

Exercise 3 (Yeti vs. Jedi)**.** We consider the following two subspaces of $\mathbb{R}^2$:

1. Are these spaces homeomorphic? Which connected components of YeTI are homeomorphic to which connected components of JEdI?
2. Are YeTI and JEdI homotopy equivalent?
3. Compute all homotopy groups of all connected components.
4. Compute $H_n(\,\cdot\,;\mathbb{Z})$ of these spaces for all $n \in \mathbb{Z}$.
5. Compute the Euler characteristic of these spaces.
6. Which connected components admit a 2022-sheeted connected covering?

Exercise 4 (summary)**.** Write a summary of Chapter 5 (Cellular Homology), keeping the following questions in mind:

1. What are typical examples of CW-complexes and cellular maps?
2. What is the geometric idea of cellular homology? What is the definition?
3. How can cellular homology be computed?
4. How can homology theories on CW-complexes be compared?
5. What consequences does this have for practical computations?
6. What is the Euler characteristic?
7. How can the Euler characteristic be computed?
8. What are typical applications of the Euler characteristic?

No submission!