Algebraic Topology: Études

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Exercise 1 (coverings of the circle)**.** Find three pairwise non-isomorphic 3-sheeted coverings of S^1 and illustrate these coverings in a suitable way!

Exercise 2 (covering maps?)**.** Illustrate the following maps in a suitable way! Which of them are covering maps and how many sheets do they have?

1. $\mathbb{R} \longrightarrow \mathbb{R}_{\geq 0}, \quad x \longmapsto x^2$
2. $\mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R}_{>0}, \quad x \longmapsto x^2$
3. $\mathbb{C} \setminus \{0\} \longrightarrow \mathbb{C} \setminus \{0\}, \quad z \longmapsto z^{2021}$
4. $S^1 \times \mathbb{R} \longrightarrow S^1 \times S^1, \quad ([x], y) \longmapsto ([x], [y])$

Exercise 3 (covering maps from group actions?)**.** Which of the following group actions are properly discontinuous? Determine the corresponding quotient spaces!

1. the action of $\mathrm{GL}_2(\mathbb{R})$ on $\mathbb{R}^2$ by matrix multiplication
2. the action of $\mathrm{SL}_2(\mathbb{Z})$ on the upper half-plane by Möbius transformations
3. the action of $\mathbb{Z}/2021$ on S^1, where $[1] \in \mathbb{Z}/2021$ acts via
$$\begin{aligned} S^1 &\longrightarrow S^1 \\ [x] &\longmapsto [x + 1/2021 \mod 1] \end{aligned}$$
4. the action of $\mathbb{Z}/2$ on $S^1 \times S^1$, where $[1] \in \mathbb{Z}/2$ acts via
$$\begin{aligned} S^1 \times S^1 &\longrightarrow S^1 \times S^1 \\ ([x], [y]) &\longmapsto ([y], [x]) \end{aligned}$$
5. the action of $\mathbb{Z}/2$ on $S^1 \times S^1$, where $[1] \in \mathbb{Z}/2$ acts via
$$\begin{aligned} S^1 \times S^1 &\longrightarrow S^1 \times S^1 \\ ([x], [y]) &\longmapsto ([x + 1/2 \mod 1], [y]) \end{aligned}$$
6. the action of $\mathbb{Z}/2$ on $S^1 \times S^1$, where $[1] \in \mathbb{Z}/2$ acts via
$$\begin{aligned} S^1 \times S^1 &\longrightarrow S^1 \times S^1 \\ ([x], [y]) &\longmapsto ([1 - x \mod 1], [y]) \end{aligned}$$

Exercise 4 (summary)**.** Write a summary of Chapter 2.2 (Divide and Conquer), keeping the following questions in mind:

1. Which types of constructions of spaces are compatible with π_1 ?
2. Which of the results carry over easily to higher homotopy groups?
3. What are the main ideas of the corresponding proofs?
4. What are the main examples?
5. What are the limits of computability of fundamental groups?

No submission!