# Algebraic Topology: Études

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**Exercise 1** (classification of coverings). Apply the classification theorem of coverings to the following spaces and give geometric description of each class of coverings:

1. $\mathbb{R}P^{2021}$
2. $S^1 \times \mathbb{R}P^{2021}$

**Exercise 2** (non-trivial coverings). Which of the following spaces admit non-trivial coverings?

1. $\mathbb{R}^2 \setminus \{0\}$
2. $\mathbb{R}^{2021} \setminus \{0\}$
3. $S^{2021} \setminus \{e_1, -e_1\}$
4. $\mathbb{R}^2 \setminus \{e_1, -e_1\}$

**Exercise 3** ("random" coverings). Let $(T, t_0) := (S^1, e_1) \times (S^1, e_1)$.

1. Roll four dice; let $a, b, c, d$ be the results and let
$$H := \operatorname{Span}_{\mathbb{Z}} \left\{ \begin{pmatrix} a-1 \\ b-1 \end{pmatrix}, \begin{pmatrix} c-1 \\ d-1 \end{pmatrix} \right\} \subset \mathbb{Z}^2.$$
2. Choose an isomorphism $\pi_1(T, t_0) \cong_{\mathsf{Group}} \mathbb{Z}^2$ and consider the subgroup $H'$ corresponding to $H$ under this isomorphism.
3. Draw the path-connected, pointed, covering of $(T, t_0)$ associated with $H'$.
4. Iterate!
5. What is the probability that the resulting total space is homeomorphic to $\mathbb{R}^2$ ?

**Exercise 4** (exact sequences). Refresh your memory of the following algebraic terms (Appendix A.6.1):

1. (short) exact sequence
2. split exact sequence
3. five lemma
4. flat module

No submission!