ℓ^1-Homology and Simplicial Volume: *Errata and Comments*

Comments and corrections for my thesis ℓ^1-*Homology and Simplicial Volume*, which is available online at

http://nbn-resolving.de/urn:nbn:de:hbz:6-37549578216,

are welcome – please send an email to clara.loeh@uni-muenster.de.

– *Proposition 2.20 on p. 25:*
  It is erroneously stated that $H_0^{\ell^1}(G;V) \cong V_G$ for all discrete groups G and all Banach G-modules V.

  A straightforward computation shows that $H_0^{\ell^1}(G;V) \cong V/U$, where

$$U = \biggl\{ \sum_{j\in\mathbf{N}} a_j \cdot (v_j - g_j \cdot v_j) \biggm| (a_j)_j \subset \mathbf{R},\ (g_j)_j \subset G,\ (v_j)_j \subset V \text{ and } \sum_{j\in\mathbf{N}} |a_j| \cdot \|v_j\| < \infty \biggr\}.$$

  We have $V/\overline{U} = V_G$, but in general U is not closed in V and so V/U need not be equal to V_G.

  If V is a reflexive Banach space, then indeed $H_0^{\ell^1}(G;V) \cong V_G$: If V is reflexive, then $0 = H_{\mathrm{b}}^1(G;V') \cong H^1(C_*^{\ell^1}(G;V)')$ [3; Propositon 6.2.1]. Therefore, $H_0^{\ell^1}(G;V)$ is Banach [2; Theorem 2.3] and hence $H_0^{\ell^1}(G;V) \cong V/\overline{U} = V_G$.

Bühler [1] provides a slightly different version of ℓ^1-homology (using the framework of exact categories) that evaluates to the coinvariants in degree 0.

- *p. 34, line −6 and Section 3.3:*
  The converse of the second part of the translation principle (Theorem 3.1) does not hold in general:

  Let $C = D$ be a Banach chain complex concentrated in degrees 0 and 1 that consists of a bounded operator $\partial \colon C_1 \longrightarrow C_0$ that is not surjective but has dense image (e.g., the inclusion $\ell^1 \hookrightarrow c_0$). In particular, the semi-norm on $H_*(C) = H_*(D)$ is zero. The morphism $f \colon C \longrightarrow D$ given by multiplication by a constant $c \in \mathbf{R} \setminus \{-1, 0, 1\}$ induces an isometric isomorphism $H_*(f) \colon H_*(C) \longrightarrow H_*(D)$.

  On the other hand, the coboundary operator $\partial' \colon C_0' \longrightarrow C_1'$ does not have dense image [4; Corollary of Theorem 4.12]. Therefore, there are elements in $H^1(D')$ of non-zero semi-norm. So $H^*(f')$, which is multiplication by c, is not isometric.
- *proof of Corollary 4.8, p. 49:*
  The third line should read
  $$H_b^*(\varphi; f') = H^*(C_b^*(\varphi; f')^G \circ i)$$
  instead of $C_b^*(\varphi; f') = H^*(C_b^*(\varphi; f')^G \circ i)$.
- *Corollary 4.10 on p. 50:*
  The statement should be corrected to: *Let G be a discrete group and let V be a Banach G-module. Then $H_*^{\ell^1}(G; V) \cong H_*^{\ell^1}(1; V_G)$ if and only if $H_b^*(G; V') \cong H_b^*(1; V'^G)$* [instead of $H_*^{\ell^1}(1; V)$ and $H_b^*(1; V')$].
- *Corollary 4.11 on p. 50:*
  Second part: $H_*^{\ell^1}(G; V) \cong H_*^{\ell^1}(1; V_G)$ instead of $H_*^{\ell^1}(G; V) \cong H_*^{\ell^1}(1; V)$.
- *Case of hyperbolic fundamental groups, p. 75:*
  The argument only applies for aspherical (or rationally essential) manifolds of dimension not equal to 1 (because Mineyev's result is concerned only with the case of dimension not equal to 1); in dimension 1 the statement is not true (the circle S^1 has hyperbolic fundamental group, but has simplicial volume equal to 0).
- *Proof of* $1 \Rightarrow 2$ *of Proposition 6.4 on p. 80:*
  In the definition of z_k the indices got messed up. Correct is:
  $$z_k := z + \sum_{j=0}^{k-1} \partial(b_j) \in C_n(M).$$

– *proof of* 1 $\Rightarrow$ 2 *of Theorem 6.1 on p. 81:*
In the last line, the correct type of q_t is

$$q_t \colon \partial W \times [0, \infty) \longrightarrow \partial W \times [0, t].$$

– *Example 6.9 on p.85f:*
In order for M to be of dimension n, we have to take N of dimension $n-2$, and hence we have to assume that $n \geq 6$ [instead of $n \geq 5$].

Bibliography

[1] T. Bühler. On the duality between ℓ^1-homology and bounded cohomology. Preprint, 2008. Available online at arXiv:0803.0680v2 [math.KT].

[2] S. Matsumoto, S. Morita. Bounded cohomology of certain groups of homeomorphisms. *Proc. Amer. Math. Soc.*, 94, no. 3, pp. 539–544, 1985.

[3] N. Monod. *Continuous Bounded Cohomology of Locally Compact Groups*. Volume 1758 of *Lecture Notes in Mathematics*, Springer, 2001.

[4] W. Rudin. *Functional Analysis*. McGraw-Hill Series in Higher Mathematics. McGraw-Hill, 1973.

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